2.6 kbar, which are similar to those indicated in [1]. One should thereby note that glass withstands rather large tensile stresses (up to 6 kbar) not only within the medium but also on the boundaries (along the ray direction), and consequently we are dealing with a material whose strength is arbitrary and is determined by defects located on the surface (a case of fracturing from an internal defect occurred in one of the 82 experiments). It is necessary for an accurate determination of the strength characteristics of a material to have even more accurate data on the stress state at the fracture point.

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CONSTRUCTION OF THE TIME DEPENDENCE OF THE RELAXATION OF TANGENTIAL STRESSES ON THE STATE PARAMETERS OF A MEDIUM

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UDC 539.3

A change in the stress state of real rigid bodies can occur not only as a result of movement of the medium but also in the absence of any macroscopic displacements of the medium at all and inflow or outflow of heat from its elements. It was proposed by Maxwell to characterize this process as a decrease of the tangential stresses, and the concept of their relaxation time has been introduced. These ideas underwent further development in [1-3], in the first of which a representation of the relaxation time τ of the tangential stresses is discussed from the molecular-kinetic viewpoint, and in the other models are formulated and analyzed of media having a nonlinear dependence of τ on the temperature and stresses. Unfortunately, methods are presently lacking for direct experimental determination of the relaxation time in the case of intensive dynamic loads. In order to determine τ, it is necessary to use indirect experimental data, including at the same time different models of a deformable rigid body. An interpolation formula of the dependence for some metals derived on the basis of information existing in the literature about the dependence of the dynamic yield point σ_T and the tensile strength limit on the deformation rate $\dot{\varepsilon}$ has been given in [4] and then refined in [5]. A dependence of the relaxation time of the tangential stresses on their strength σ, the values of the shear (plastic) strain, and the temperature T is constructed in this paper on the basis of dislocation concepts concerning the mechanism of the relaxation process with the inclusion of a model of a viscoelastic body [3] and experimental data similar to that used in [4, 5].

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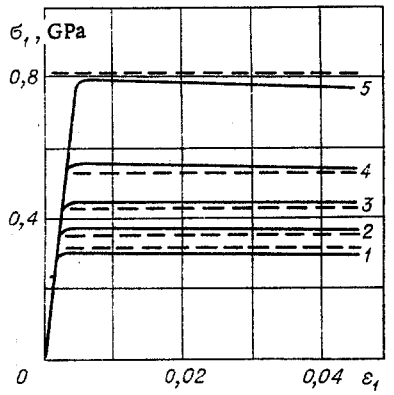

Fig. 1

Either the metric strain tensor g_{ij} or the Almansi strain tensor $\varepsilon_{ij} = {}^1/_2(\delta_{ij} - g_{ij})$, where δ_{ij} is the Kronecker symbol, is used in [3]. We will restrict ourselves to consideration of small strains, in which case the Almansi tensor coincides with the tensor of small strains introduced in the traditional way. We will represent the strain tensor in the form of a sum of elastic e_{ij} and plastic s_{ij} strain tensors

$$\varepsilon_{ij} = e_{ij} + s_{ij}. \tag{1}$$

Within the framework of this paper it is sufficient to restrict oneself to consideration of the problem of the strain of a circular rod. The cylindrical coordinate system which is a natural choice for this problem coincides with the principal axes of the strain tensor, which, like the stress tensor, takes on a diagonal form in this case and is characterized by the components ε_i, i = 1, 2, 3. Equation (1) takes the form

$$\varepsilon_i = e_i + s_i.$$

The maximum plastic strain s (the principal value of the plastic strain tensor) is determined in terms of s_i:

$$s = (s_1 - s_3)/2 = (s_1 - s_3)/2 = 3s_1/4 = -3s_2/2$$

and occurs in the plane of maximum tangential stress. Since relaxation of the tangential stresses is completed in the process of plastic strain, it is natural to assume that the relaxation time is proportional to the inverse of the maximum plastic strain rate:

$$\tau \sim 1/(ds/dt).$$

Plastic strain is accomplished by means of the motion of dislocations, and its rate is a function of the characteristics of the system of moving dislocations as given by the Orovan relation:

$$ds/dt = bNv,$$

where N and v are the density of fixed dislocations per unit volume and their average velocity and b is the modulus of the Burgers vector, whence

$$\tau \sim 1/(bNv). \tag{2}$$

The averaged characteristics of an ensemble of dislocations N and v are in turn functions of the strength of the tangential stresses σ, the value of the plastic strain s, and the temperature T.

Depending upon the level of the temperature and stresses caused by the external load, motion of the dislocations can take place in three modes which differ qualitatively from each other. When the loads are relatively small in comparison with the yield point of the material, creeping of the dislocations occurs — a phenomenon which is of a diffusion nature and which proceeds at the rates of diffusion processes. The behavior of materials under such conditions, including relaxation of tangential stresses, has been described within the framework of models of creep theory. If the strength of the tangential stresses is close to the static yield point, the rate of movement of the dislocations is determined by thermoactivation of transverse glide of the dislocations — the region of so-called thermoactivated subbarrier glide. As the external stresses are increased further, continuous suprabarrier glide of the dislocations occurs, by means of which the final plastic strains of metals are realized.

TABLE 1

Fe	T, K	195	300	493	713	1100	1300	1500
	lg a	−2,93	−5,64	−8,94	−11,92	−0,17	−1,44	−0,39
	σ_0, GPa	5,45	5,30	5,35	5,32	0,64	0,5	0,1
	δ, %	2,5	11	9,4	11,1	15,1	26,4	29,7

Cu	T, K	400	600	800	1000	Al	T, K	300	500	900
	lg a	−6,3	−4,35	−3,17	−2,94		lg a	−4,43	−2,56	−2,87
	σ_0, GPa	2,0	1,2	0,64	0,32		σ_0, GPa	1,37	0,671	0,08
	δ, %	5,5	15	7	9		δ, %	13,2	1	3

The last mode of dislocation movement, which has been least investigated experimentally and theoretically, is the most interesting from the point of view of the investigation of the relaxation processes of tangential stresses, since it corresponds to the region of maximally steep nonlinear variation of the time dependence of relaxation on the state parameters of a medium. The main experimental results which are the basis for construction of model descriptions of the process of movement and multiplication of dislocations are obtained in [6]. Here the maximally broad region of variation of the rates of movement of dislocations (right up to 1 km/sec) has evidently been investigated for stresses much greater than the static yield point of the material. As a result of the interpretation of the experimental data, expressions are obtained for the average rate of movement of dislocations

$$v = v_0 \exp\left[-(\sigma_0 + Hs)/\sigma\right] \tag{3}$$

and the number of stationary dislocations

$$N = N_0 + Ms \tag{4}$$

where v_0 is the maximum possible rate of movement of the dislocations, which is equal to the velocity of shear waves; σ_0, characteristic deceleration stress; H, coefficient of strain reinforcement; N_0, initial density of stationary dislocations; and M, multiplication coefficient. Equations (3) and (4) are used in [7, 8] to explain the phenomenon of the damping of the amplitude of the elastic precursor and to describe the behavior of metals when impacted. The good agreement obtained between the experimental and calculated data provides a basis for using (3) and (4) as the fundamental expressions of a theory of dislocation dynamics.

Another form of the interpolation functions has been used in [9] in connection with an investigation of the profile of viscoelastic waves in aluminum:

$$N = N_0[1 + (Qs/bN_1)] \exp(-\lambda s); \tag{5}$$

$$v = v_0\delta^n/(1 + \delta^n),\ \delta = (\sigma - \sigma_s)/\sigma_k, \tag{6}$$

where N_1 is the initial density of all dislocations; σ_s, static yield point of the material; and Q, λ, σ_k, and n, some constants associated with the initial state of the material. The use of these dependences also leads to good agreement of the calculated and experimental data.

Equations (3) and (4) or (5) and (6) together with (2) permit establishing the form of the function which relates the relaxation time τ to the parameters characterizing the stress–strain state of a medium. We will assume that

$$\tau = a/(bNv), \tag{7}$$

where a is a coefficient of proportionality still subject to determination. One should note that the parameters entering into Eqs. (3)-(6) and the coefficient a depends on the temperature, which permits constructing the function $\tau(\sigma, s, T)$.

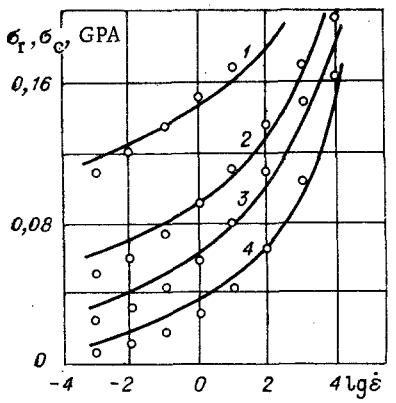

Fig. 2

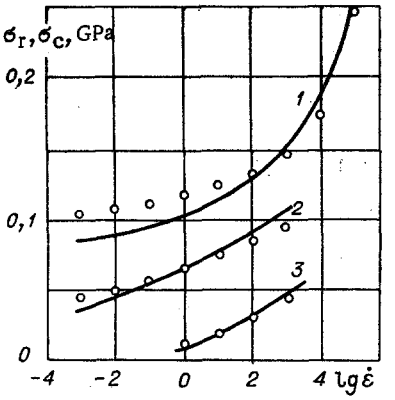

Fig. 3

Direct determination of the quantities appearing in the formulas which describe the dynamics of dislocations is a complicated experimental problem. At present reliable data on their values are known only for Armco iron [7, 8] and a single brand of aluminum [9]. These parameters have been calculated in [10] on the basis of experimentally determined static dependences of the stresses on the strain. The possibility of using the parameters found in this way in the case of deformation at high rates remains unclear.

We will determine the parameters appearing in (7) (this means in (3)-(6)) on the basis of "numerical experiments" on the uniaxial strain of a thin rod. Formulation of the problem in a model of a Maxwellian-type viscoelastic body was discussed in [5, 11]. The problem reduces to the solution of the system of equations

$$\begin{aligned} d\alpha/dt &= \dot{\varepsilon} - (\alpha - d)/\tau, \\ de/dt &= 4c^2[(\alpha - d)^2 + (\beta - d)^2 + (\gamma - d)^2]/2T\tau. \end{aligned} \tag{8}$$

where $d = (\alpha + \beta + \gamma)/3$; α, β, and γ are the logarithms of the relative expansions along the longitudinal and transverse coordinate axes, respectively, c is the propagation velocity of transverse waves, and e is the entropy density per unit mass. The system (8) is supplemented by the equation of state of the material in the form of the dependence of the internal energy density $E = E(\alpha, \beta, \gamma, e)$, by the Murnaghan formulas

$$\sigma_1 = \rho\partial E/\partial\alpha,\ \sigma_2 = \rho\partial E/\partial\beta,\ \sigma_3 = \rho\partial E/\partial\gamma,\ T = \partial E/\partial e \tag{9}$$

(σ_i (i = 1, 2, 3) are the stresses and ρ is the density), and by the relationships

$$\sigma_2 \equiv \sigma_3 = 0,\ \beta \equiv \gamma. \tag{10}$$

As a result of the solution of the problem (8)-(10) with specified ε = const and $\tau = \tau(\sigma, s, T)$ $\sigma_1(\varepsilon_1)$ — the stress—strain diagram of the material — is determined, which one can compare with the appropriate experimental dependence.

Due to the presence on the right-hand side of the first equation of the function $\tau(\sigma, s, T)$, which varies sharply over a large range of values, system (8) is a rigorous autonomous system of ordinary differential equations. The ratio of the maximum and minimum eigennumbers of its Jacobian matrix is of the order of $\sim 10^3$. The application of explicit methods of the Runge—Kutta and Adams type for the solution of such systems is made more difficult due to the strong restriction on the integration step over the entire computational interval, which is caused by the requirement of stability of the numerical solution. Special efficient methods of solution which permit significantly reducing the computation time are being developed [12] for the solution of rigorous systems. The method of variable order and step constructed in [13] on the basis of A-stable Rosenbrock methods is used in this paper.

Let a rigorous system of K ordinary differential equations be given with the initial data

$$dy/dt = f(y),\ y(t_0) = y_0,\ y = (y_1, \ldots, y_K).$$

The general form of the Rosenbrock formulas applicable to this system is as follows:

$$y_{n-1}^{(m)} = y_n + \sum_{i=1}^{m} p_i^{(m)} k_i, \quad k_i = h_n [I - h_n a_i A(\xi_i)]^{-1} f(\eta_i),$$

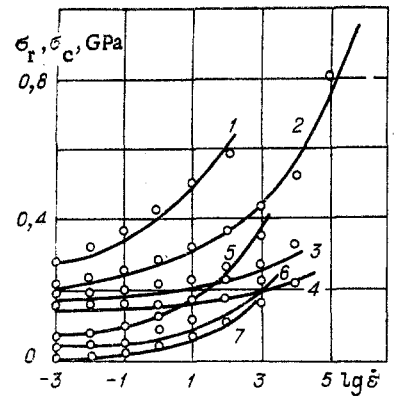

Fig. 4

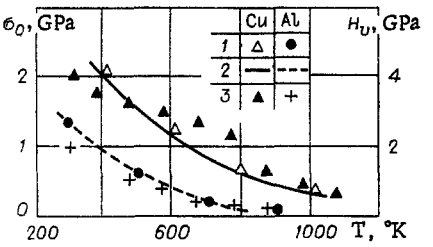

Fig. 5

$$\xi_i = y_n + \sum_{j=1}^{i-1} \lambda_{ij} k_j, \quad \eta_i = y_n + \sum_{j=1}^{i-1} \mu_{ij} k_j \quad (i = 2, \ldots, m),$$

$$\xi_1 = \eta_1 = y_n,$$

where $A(z) = \partial f/\partial y|_{y=z}$ is the Jacobi matrix of the system under discussion, $y_{n+1}^{(m)}$ is the value of the function calculated by the m-stage method for time t_{n+1}; y_n and h_n, value of the function and the integration step at time t_n; and I, unit matrix with size K × K. The coefficients $p_i^{(m)}$, a_i, λ_{ij}, and μ_{ij} are chosen such that the m-stage method has an order of approximation equal to m, is A-stable, and the number of calculations and inversions of the Jacobi matrix in a single integration step is a minimum. The numerical values of the coefficients of the method for the case m = 4 are given in [13].

The local error of the approximation is determined from the formula

$$\delta y_n^{(m)} = \min_{m<j\leqslant 4} \| y_n^{(j)} - y_n^{(m)} \|, \quad m = 1, 2, 3,$$

where $\| \|$ is the maximum norm of the vector. The integration step for time t_{n+1} is chosen so that the inequality

$$\delta y_{n+1}^{(m)} \leqslant \varepsilon (\varepsilon/c_m)^{1/m}, \quad m = 1, 2, 3, \tag{11}$$

is satisfied, where ε is the required accuracy of the solution and c_m are some constants given in [13]. Prediction of the integration step is done on the basis of the inequality

$$\delta y_{n+1}^{(m)} \leqslant \varepsilon (\varepsilon/c_m)^{1/m} q^{i(m+1)}, \tag{12}$$

which is obtained in connection with the investigation of the local approximation error. If this inequality is satisfied for some i and q (i is a natural number, q is a real number), then the integration step can be increased by a factor of q^i. When (11) is not satisfied for any values of m, it is necessary to decrease the step by a factor of $q^{|i|}$, where i is the maximum integral negative number for which (12) is valid. The same m-stage method for which the maximum step is predicted is used for the calculations at the next step. When (12) is satisfied, the value $y_{n+1}^{(m)}$ is taken for the value of the desired function at the point $t_{n+1} = t_n + h_n q^i$.

The method outlined with $m \leqslant 4$ was used to solve the system (8)-(10). The algebraic equation which enters into this system was solved at each time step by Newton's method. The dependence of the elastic energy on the compression parameters in the case of a nonspherical strain tensor which is given in [14] was used in solving the problem. The computational results for iron at room temperature with Eqs. (3) and (4) and the values of the parameters appearing in them which are given in [8] are shown in Fig. 1 in the form of the dependence $\sigma_1(\varepsilon_1)$. The quantity α in (7) was chosen by the method outlined above. Curves 1-5 corresponds to values of 10, 10^2, ..., 10^5 for the strain rate. The form of the dependences obtained is characteristic for an ideal elastoplastic medium: there is a section of linear dependence between stress and strain and a section of an increase of the strain with an inappreciable change in the stress. Some decrease of the stress with an increase in the strain is explained by stress relaxation due to heating of the medium upon plastic strain. Comparison of the calculated values of the dynamic yield points with the experimental data for

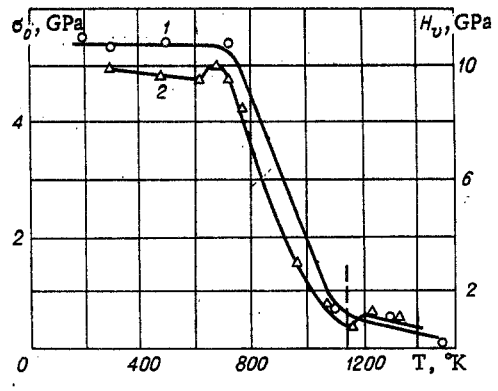

Fig. 6

soft steel [15] (they are given in Fig. 1 in the form of dashed lines) shows their good agreement, notwithstanding the difference in the brands of iron used in [8, 15].

Now we will consider the problem of finding the parameters which enter into Eq. (7). One can obtain a number of calculated values of the dynamic yield point σ_{ci} by conducting numerical experiments on the uniform deformation of a thin rod at different fixed strain rates. Due to the peculiarities of the computational model selected, these values will depend on $\dot{\varepsilon}$ and the values of the parameters in the equations which determine the dynamics of dislocations (σ_0, H, N_0, and M in (3) and (4) and N_0, Q, λ, σ_k, and n in (5) and (6)), as well as the values of α in (7). Having denoted the set of all these parameters as $r_1, \ldots, r_k$, where k is equal to 5 or 6, depending upon the equations used, and taking account of the fact that the values of r_i themselves (i = 1, ..., k) vary as the temperature at which the experiment is performed changes, one can write that $\sigma_{ci} = \sigma_{ci}(\dot{\varepsilon}_i, r_1(T), \ldots, r_k(T))$. Now if a set of the actual values of the yield point $\sigma_{ri}(\dot{\varepsilon}_i)$, i = 1, ..., F is known for a given material and temperature from a physical experiment, then we introduce the function

$$R(r_1(T), \ldots, r_k(T)) = \sum_{i=1}^{F} [\sigma_{ci}(\dot{\varepsilon}_i, r_1, \ldots, r_k) - \sigma_{ri}(\dot{\varepsilon}_i)]^2,$$

which determines the sum of the squares of the deviations of the experimental values of the yield points from the calculated values. It is natural to assume those values for them which yield a minimum of the function R as the solution of the problem of finding the parameters r_i (i = 1, ..., k) in Eq. (7) for the relaxation time, if this minimum is sufficiently close to zero. The gradient descent method was used to find the minimum of R(r), where r is a vector with the components $(r_1, \ldots, r_k)$. Let r_j be the value of r obtained in the j-th approximation; then

$$r_{j+1} = r_j - h_0 \operatorname{grad} R(r_j),$$

where h_0 is determined from the condition

$$R(r_j - h_0 \operatorname{grad} R(r_j)) = \min_{h>0} R(r_j - h \operatorname{grad} R(r_j)).$$

The following computational algorithm was used in implementing this method on a computer: let us introduce the notation $f(h) = r_j - h \operatorname{grad} R(r_j)$; then if $R(f(h)) < R(r_j)$, $r_{j+1} = f(2^g h)$, where g is the maximum natural number for which the inequality

$$R(f(2^g h)) < R(f(2^{g-1} h))$$

is valid. If $R(f(h)) \geqslant R(r_j)$, $r_j = f(h/2^\omega)$, where ω is the minimum natural number for which $R(f(h/2^\omega)) < R(r_j)$, is taken as the next approximation. The calculation is finished either when $|R(r_j)| < \varepsilon^0$ or when $|R(r_{j+1}) - R(r_j)| < \varepsilon_0$, where ε^0 and ε_0 are some small quantities specified in advance.

It is necessary in connection with the solution of the problem of finding the parameters which enter into (7) to minimize R(r) for 5-6 variables. As an analysis specially performed on the basis of the computational results shows, variation of the various parameters in (7) affects the value of the calculated yield point in different ways. Thus variation of the initial number of fixed dislocations in the range 10^5 cm$^{-2} \leqslant N_0 \leqslant 10^9$ cm^{-2} with fixed values of the remaining parameters and $\dot{\varepsilon}$ results in a change in σ_{ci} by 2-3% in all. This result can be explained by taking account of the difference between the values N_0 and M ($M/N_0 \sim 10^4$-10^5),

TABLE 2

Metal	T, °K	lg a	lg(Q/N_1)	σ_k, GPa	n	λ	δ, %
Al	300	—9,4	—5,85	0,28	4,73	0	14
Fe	195	—4,9	—4,35	0,28	4,73	0	8
Cu	300	—7,7	—4,65	0,47	4,48	0	9,4

from which it follows that the initial number of fixed dislocations determines the value of the relaxation time only in a small neighborhood of the discontinuity, when the shear strain is very small. One can convince oneself from similar discussions that the rate of movement of the dislocations is determined by the value of the characteristic stress. This parameter will evidently be most sensitive to a temperature change, by virtue of its direct connection with the value of the initial stress, and consequently with the height of the peaks of the potential relief of the crystalline lattice of the material. Taking account of the fact that (as indicated in [16, 17]) a variation of the temperature of the sample has no appreciable effect on the density of the dislocations responsible for plastic strain, one should assume that the main parameters to be varied in (7) should be $\sigma_0(T)$ and $a(T)$ in the case of Eqs. (3) and (4) or their analog when (5) and (6) are used. Numerical calculations have shown that the dependences $\sigma_c(\dot{\varepsilon})$ obtained on the basis of the solution of the problem of minimization of the complete set of parameters and the two indicated practically coincide.

It was not proven in the solution of this problem that R(r) has a single minimum or that the minimum found is an absolute one. The reliability of the parameter values obtained was estimated from the standpoint of their physical plausibility. Numerical experiments in which the initial values of the parameters being sought were varied over a physically realistic range for a number of temperatures have shown that the minimum found in such a region is unique.

The data of the computations of the parameters $a(T)$ and $\sigma_0(T)$ are given in Table 1 for aluminum, copper, and iron. The maximum differences between the experimental $\sigma_r(\dot{\varepsilon})$ and calculated $\sigma_c(\dot{\varepsilon})$ dependences δ are also indicated here for each case. These differences, which are sometimes rather large, refer most often to one of the values being compared and may be a consequence both of an inexactness of the discussion which has been carried out and a result of experimental scatter of the quantities. The most accurate representation of the differences between the calculated dependences and the experimental data gives the comparison of them over the entire range of variation of $\dot{\varepsilon}$ which is shown in Fig. 2 (copper, curves 1-4 correspond to T = 400, 600, 800, and 1000°K), Fig. 3 (aluminum, 1-3 correspond to T = 300, 500, and 900°K), and Fig. 4 (iron, 1-7 correspond to T = 195, 300, 493, 713, 1100, 1300, and 1500°K); the experimental values σ_r are indicated here by open circles. As follows from Figs. 2-4, the calculations give a rather good description of the experimental dependences $\sigma_r(\dot{\varepsilon})$.

Whereas $a(T)$ is a purely fitting parameter, a definite physical meaning is assigned to the quantity σ_0 upon construction of the dependence (3). In this connection it appears necessary to analyze the dependences $\sigma_0(T)$ found. An expression relating the characteristic deceleration stress with the temperature has been obtained in [18] on the basis of an analysis of the interaction of dislocations with point defects:

$$\sigma_0(T) = B \exp(-T/\theta)/T, \tag{13}$$

where θ is a constant coefficient and B is a constant which depends on the lattice parameters, the dislocation branch, and the concentration of defects in the material. The validity of (13) has been verified in [19]; it is true for only one material and a relatively narrow range of low temperatures. Upon the appropriate choice of the quantities B and θ the calculated values of $\sigma_0(T)$ in the cases of aluminum and copper are described well by the dependence (13). A comparison is given in Fig. 5, where 1 corresponds to the calculated values of $\sigma_0(T)$, and 2 corresponds to the values calculated from Eq. (13). In the case of iron the calculated dependence $\sigma_0(T)$ differs qualitatively from the preceding ones and (13) in that it maintains practically a constant value over a wide range of low temperatures. As the temperature increases, the dependence $\sigma_0(T)$ decreases sharply as $T \approx 700$°K is reached. The sharp drop continues until temperatures close to the temperature of the α-γ phase transition are reached after which a change in the nature of the dependence occurs and the quantity σ_0 decreases

TABLE 3

Metal	E, GPa		ν	
	calc.	exp.	calc.	exp.
Al	65	68,5	0,36	0,36—0,37
Fe	171	180	0,32	0,31
Cu	114	112	0,37	0,36—0,39

more smoothly as T increases (point 1 in Fig. 6). At first glance it may appear that such a nature for the dependence indicates that it has been incorrectly calculated. However, it turns out that all the calculated dependences $\sigma_0(T)$ both for iron and for copper and aluminum repeat qualitatively the temperature dependence of a strength characteristic of the materials such as the hardness. These dependences (Vickers hardness H_V) taken from [20, 21] are also given in Fig. 5 (points 3) and Fig. 6 (points 2).

One can understand the cause of the nonobvious correlation of the dependences $\sigma_0(T)$ and $H_V(T)$ by recalling the physical meaning embodied in the concepts of hardness and the characteristic deceleration stress. Hardness has the physical meaning of the average contact stress on the surface of an impression in the process of its formation [22] and thus characterizes the resistance of the material to elastoplastic strain. Special investigations show that hardness is very sensitive to changes in the defect structure of a material. The deceleration stress characterizes the resistance to suprabarrier glide of dislocations produced by the existence of potential relief of the lattice and the presence of impurities and other types of defects in the crystalline structure. Since plastic flow is determined in the first place by the motion of dislocations, the characteristic deceleration stress is also a certain characteristic of the resistance to plastic deformation. Of course, the characteristics under discussion are obtained upon the realization of different stress—strain states; therefore their numerical values differ significantly.

A consequence of the established connection between $\sigma_0(T)$ and $H_V(T)$ is the possibility of the use of Eq. (13) with appropriately selected constants for the description of the temperature dependence of the hardness in those cases in which it is valid for the characteristic deceleration stress. One should note that Eq. (13) itself, which is obtained under very strong restrictions and for a completely specified process, is not valid for all materials and deformation conditions.

The results of calculations performed on the basis of Eqs. (5) and (6) are given in Table 2. These calculations were performed mainly for the purpose of clarifying the applicability of (5) and (6) to the description of the process of relaxation of tangential stresses. They showed that these relationships provide a correct description of the behavior of materials under the conditions being discussed. Nevertheless, the use of Eqs. (3) and (4) seems more preferable, since they have a more rigorous foundation and permit Eq. (7) to be constructed with a smaller number of fitting parameters.

A calculation of the complete σ_i—ε_i diagram of a given material under the conditions of uniaxial tension (see Fig. 1) has been performed in the course of the solution of the problem of the stretching of a rod. This fact permits determining not only the plastic but also the elastic characteristics of a metal. A comparison of the calculated and experimental values of the Young modulus E and the Poisson coefficient ν is given in Table 3. The values are found to be in good agreement, which indicates once more the applicability of the construction performed to the solution of problems of dynamic deformation of metals.

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